REI Project: Future Electric Power System in a Carbon Constrained World

Distributed vs. Centralized Planning and Trade-off Analyses for Electric Power Systems in a Carbon Constrained World

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Rutgers, The State University of New Jersey

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Study Problem Statement

- Demand for electricity is systematically increasing for an aging system of transmission line, stations, sub-stations, etc.
- Power system network expansion and utility operations must evolve to reduce greenhouse gas emissions
- Trade-offs between cost, reliability and emissions (CO_2, SO_2, NO_x) must be explicitly considered as part of any expansion plan
- Distributed power generation can offer distinct benefits
- Analytical planning and optimization tools are required for modeling and planning – our contribution!

Project Team

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Participants: David Coit, Hatice Tekiner

We acknowledge the support of the Rutgers Energy Institute

Power System Planning & Trade-off Studies are Narrowly Focused

- Reliability
  - Determination of unmet demand, loss of load probability, standard metrics (CAIDI, SAIFI)
  - Tools: historical data, simulation, stochastic models

- Expansion
  - Determination of plans:
    - where to add power generation & transmission capacity
    - what technology (coal, solar, wind, etc.)
    - time horizon for expansion
  - Tools: engineering economics studies, mathematical programming

- Operations
  - Power generation dispatching in response to demand and availability of generating units & transmission
  - Tools: standard dispatching rules, mathematical programming

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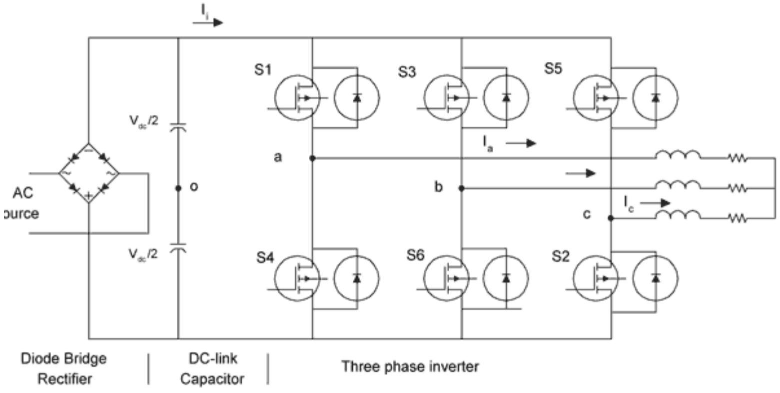

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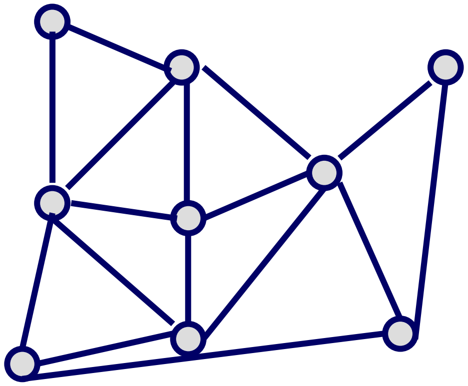

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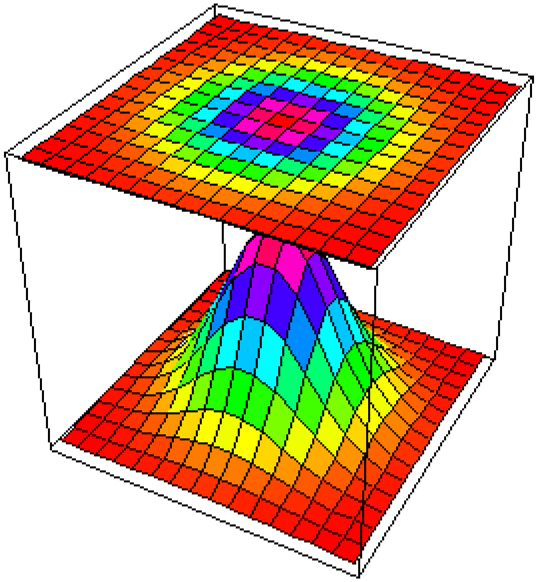

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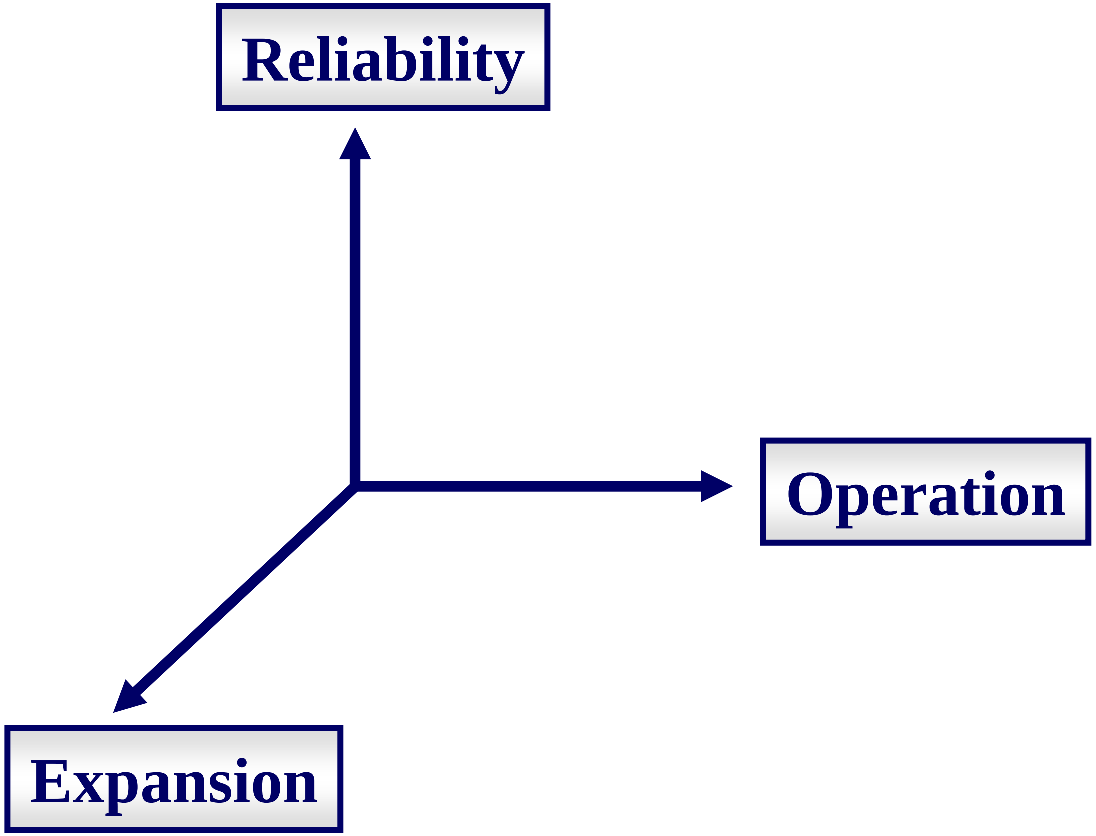

Reliability

Operation

Expansion

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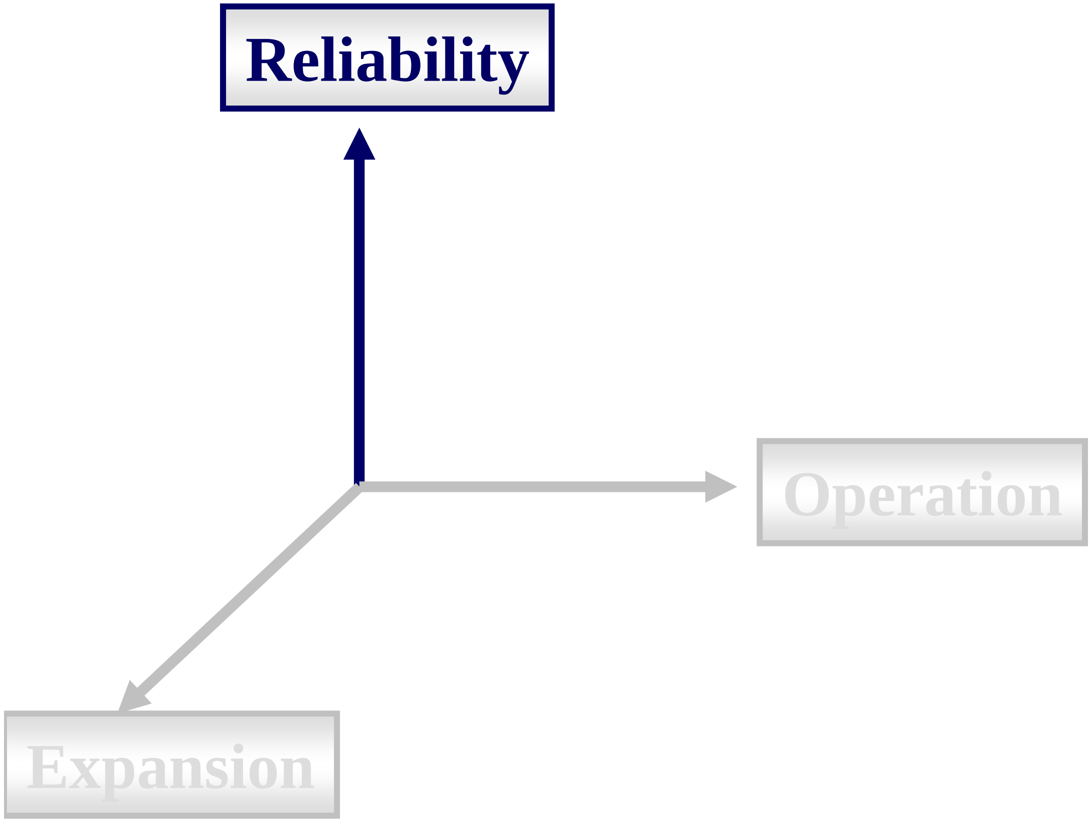

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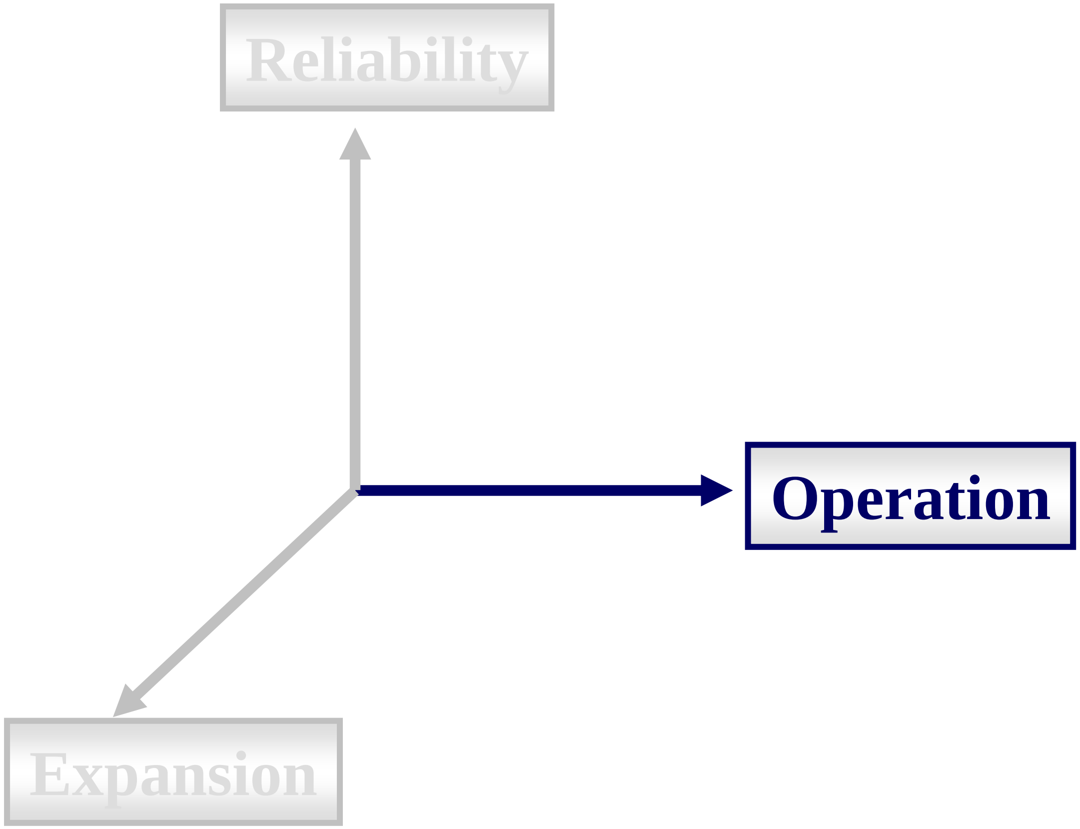
Power System Planning & Trade-off Studies are Narrowly Focused



Reliability

Operation

Expansion

- Optimal environmental dispatching of electric power systems via an improved Hopfield neural network model, King, T.D., El-Hawary, M.E., and El-Hawary, F., *IEEE Transactions on Power Systems*, Vol. 10, 1995
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- An application of Lagrangian Relaxation to Sceduling in Power Generation Systems, Muckstadt, J.A., Koeing, S.A., *Operations Research*, Vol.25, 1977
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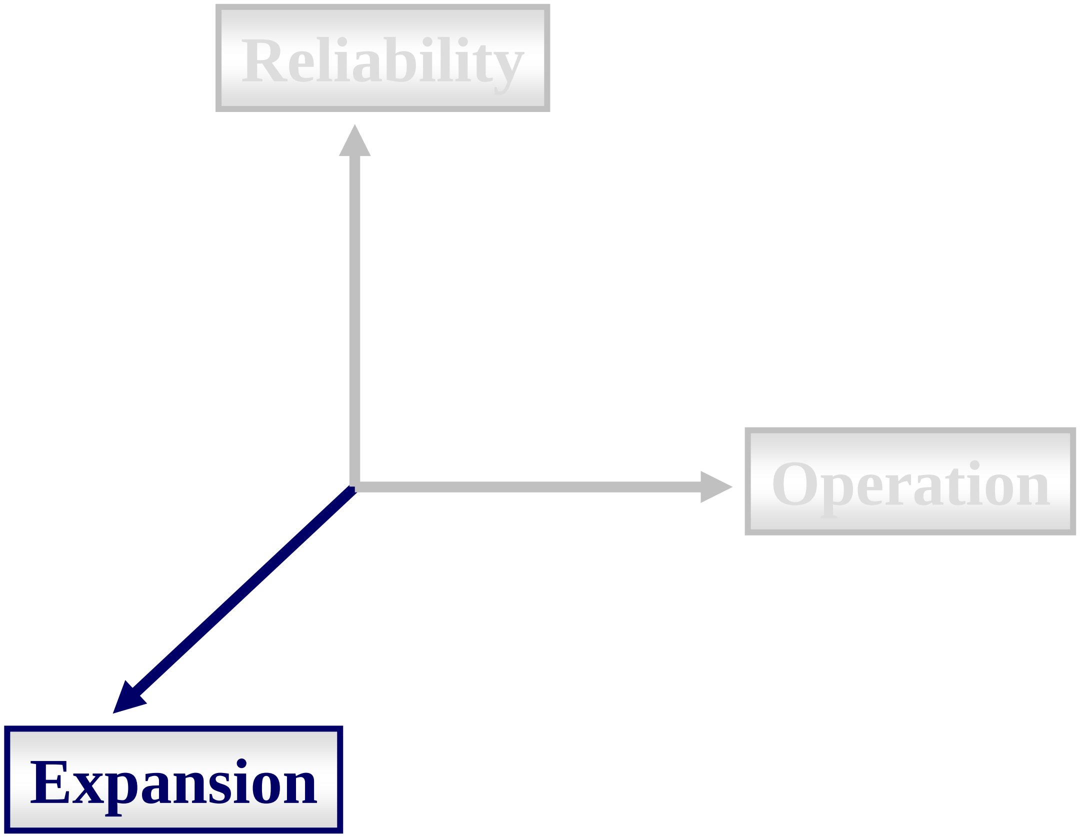

Reliability

Operation

Expansion

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- A model for Multiperiod Multiobjective Power Generation Expansion Problem, Meza, J.L.C., Yildirim, M.B., Masud, A.S.M, *IEEE Transactions on Power* Systems, Vol.22, 2007
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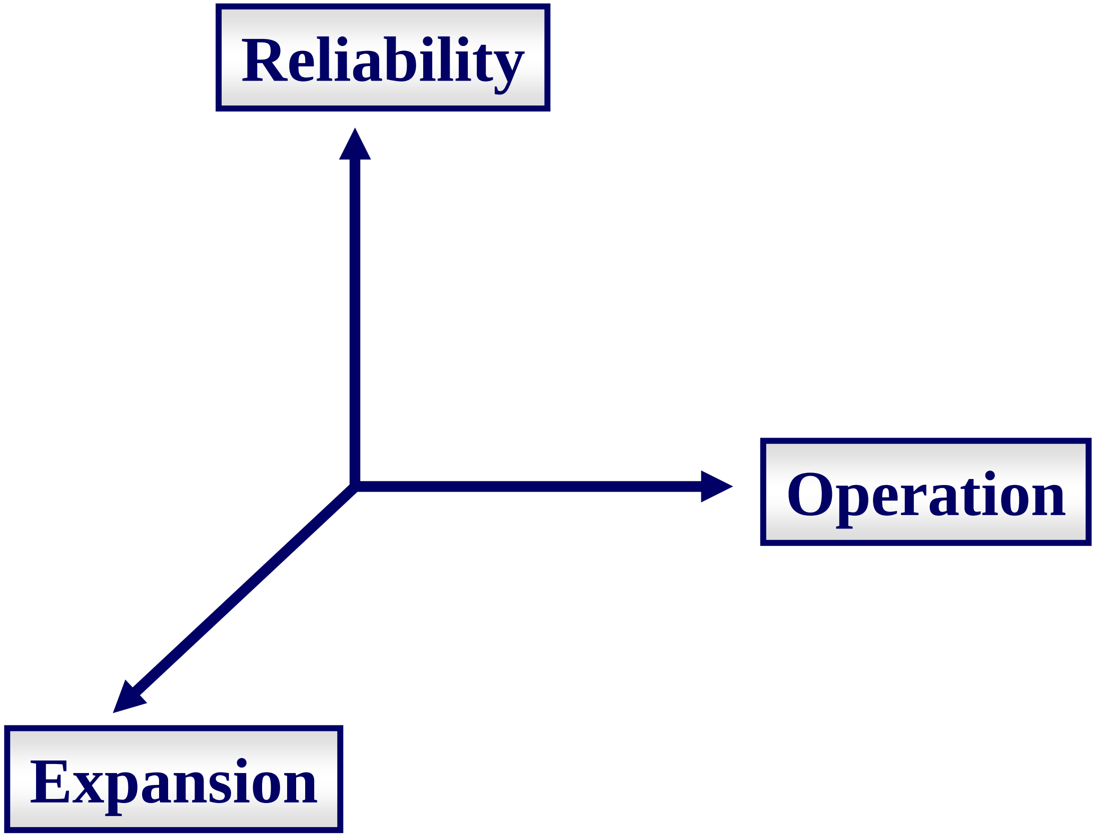


Reliability

Operation

Expansion

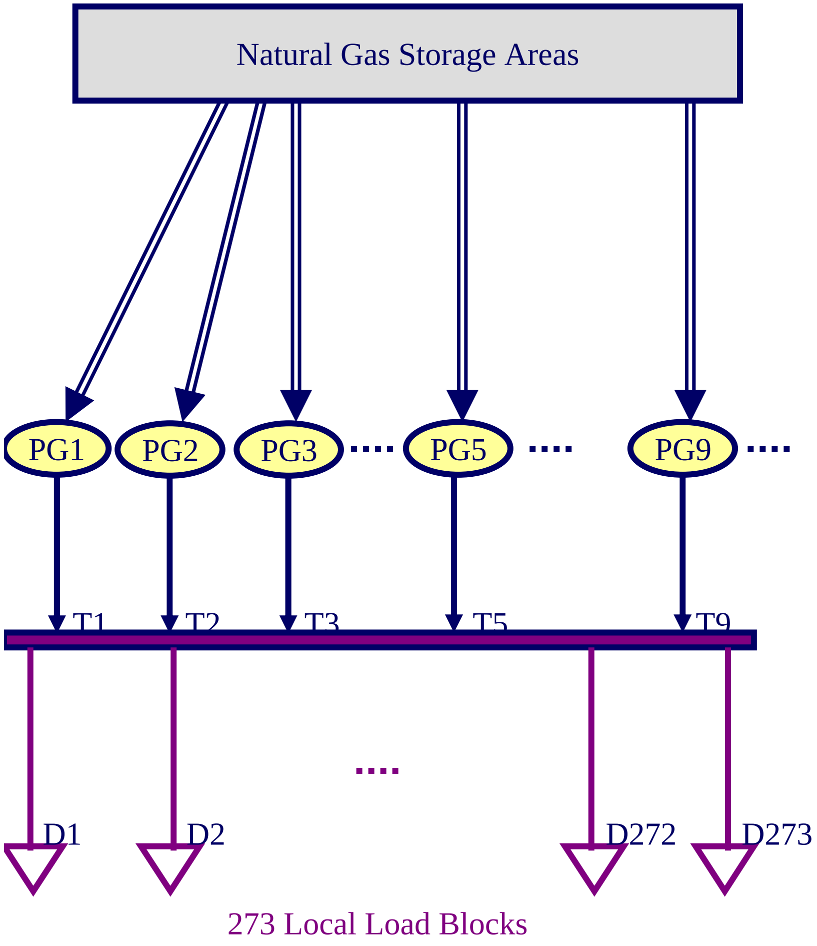
Our work integrates these dimensions simultaneously

OUR APPROACH

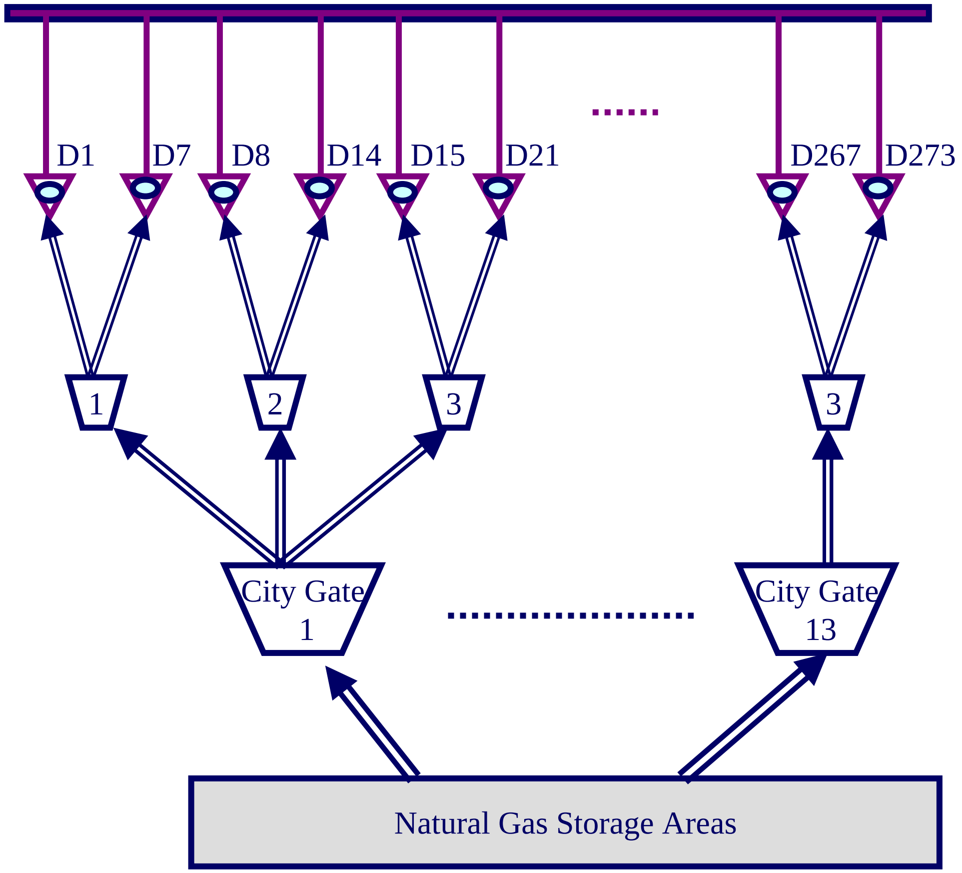
- Integrated reliability/ expansion/operations analysis
  - Considering long term planning horizon
  - Creating scenarios by Monte Carlo Simulation
- Multi-objective models
  - min Cost
  - min Greenhouse Gas Emissions (min CO_2)
  - min Other Emissions (min SO_2, min NO_X)
  - Compromise between them
- Stochastic optimization approach
  - Optimization based on power system component availability and reliability

Centralized vs. Distributed

Centralized



Distributed

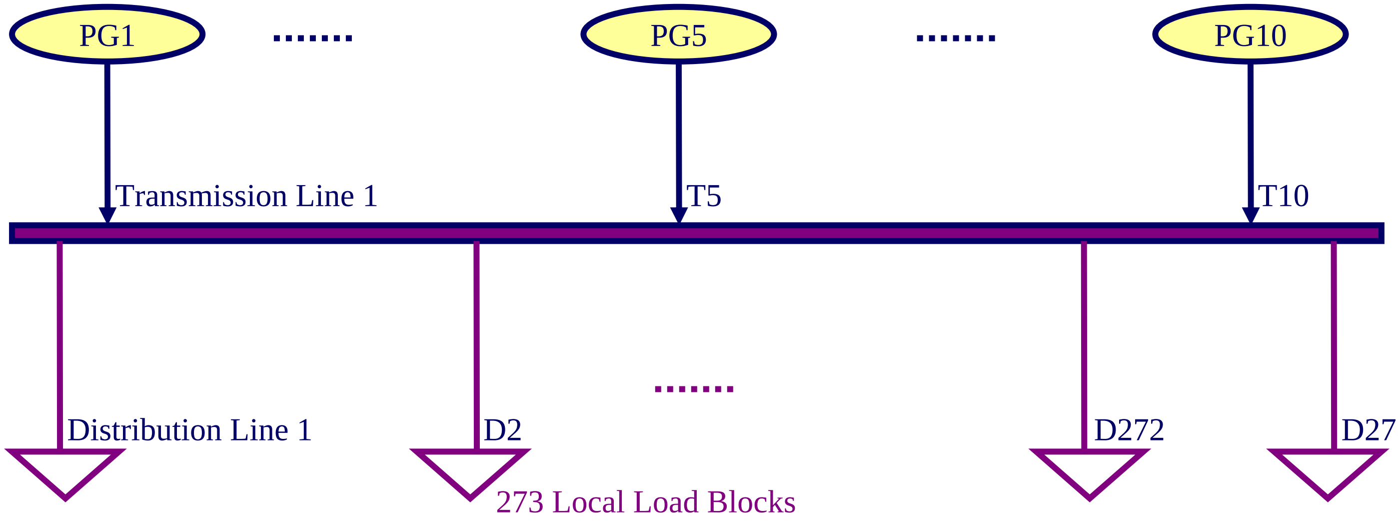


Technology Choices for Distributed Power Generation

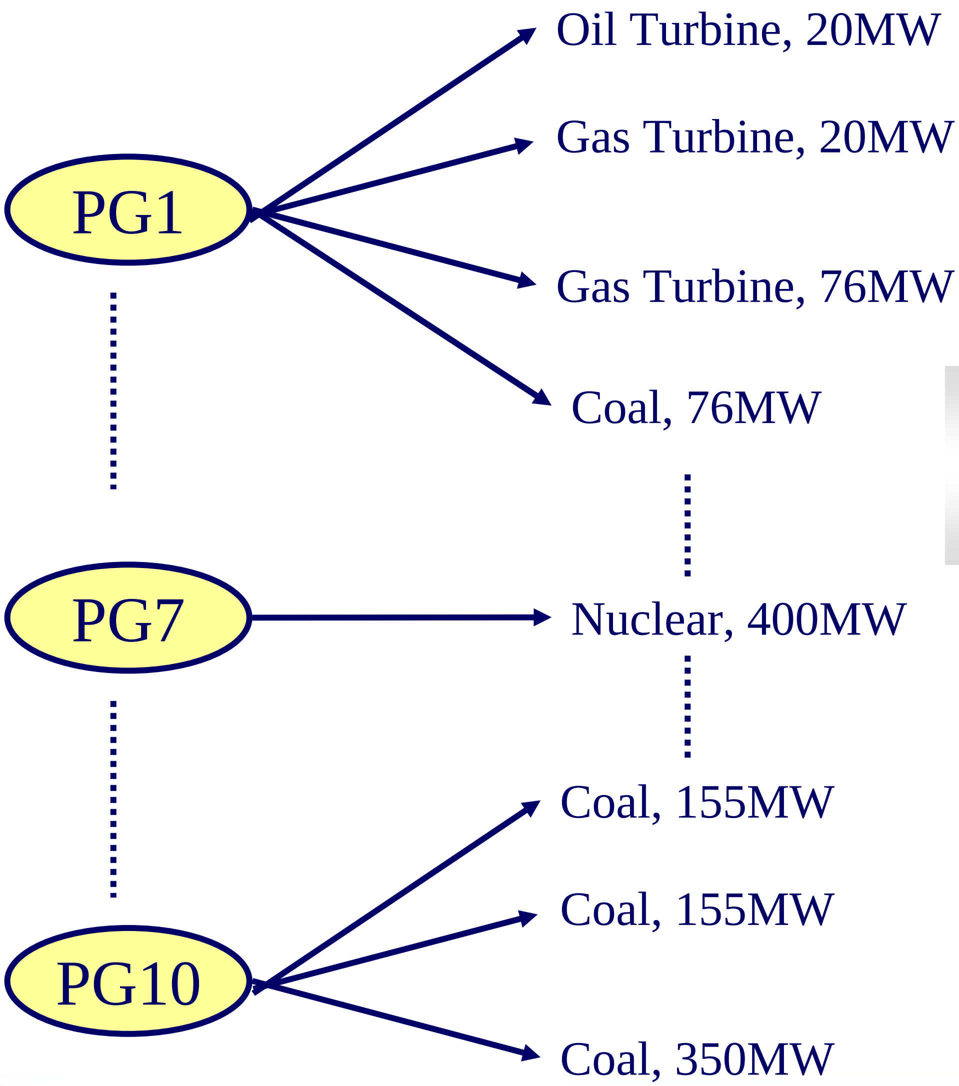
- Small wind power systems
- Photovoltaic cells - uses solar cells to convert light into electricity
- Fuel cells - electrochemical energy conversion device
- Turbines - extracts energy from a flow of hot gas produced by combustion of gas or fuel oil in a stream of compressed air
- Internal combustion engines

Centralized Test System Topology

- Our preliminary model has been successfully applied
- Test system is an adopted version of an IEEE standard test system
- System is presented in “Incorporating stress in electric power systems reliability models”, Zerriffi, H., Dowlatabadi, H., Farrel, Alex, *Energy Policy*, Vol, 35, 2007
- Test system has 10 power groups and 273 local load blocks



Power Groups in Centralized System



PG1

Oil Turbine, 20MW

Gas Turbine, 20MW

Gas Turbine, 76MW

Coal, 76MW

PG7

Nuclear, 400MW

PG10

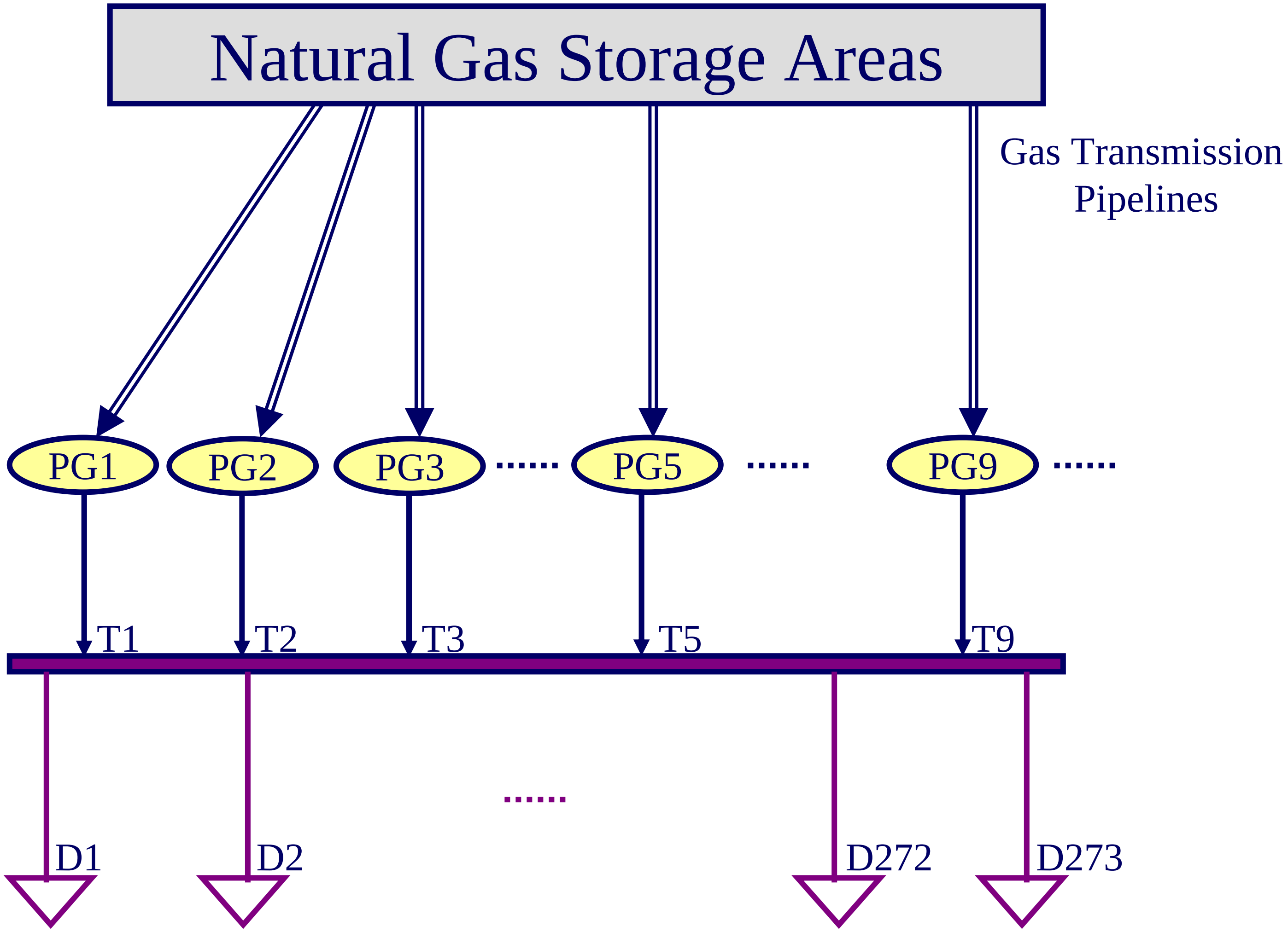
Coal, 155MW

Coal, 155MW

Coal, 350MW

Each works independent from each other

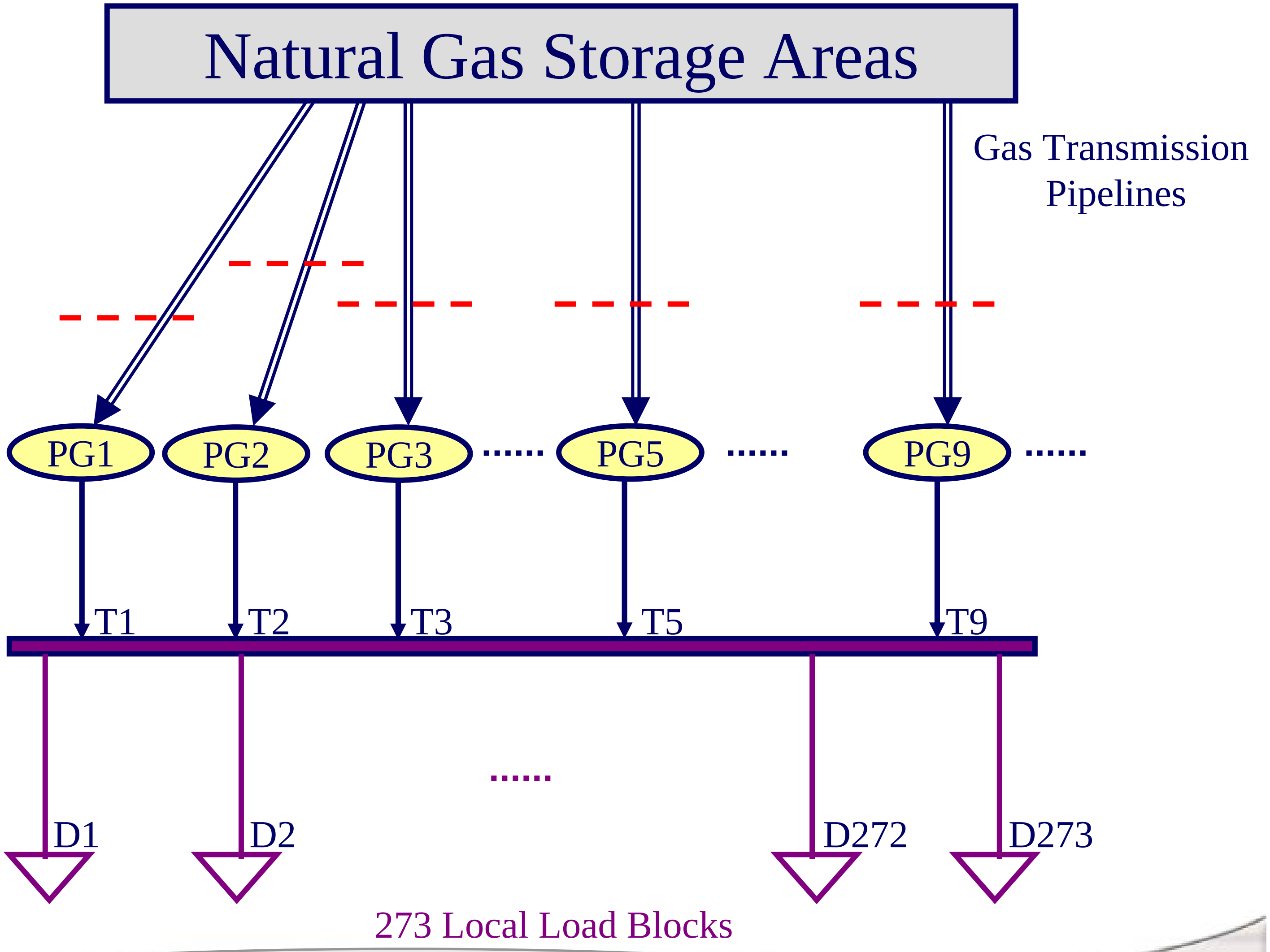
Failures Consider in Centralized Test System



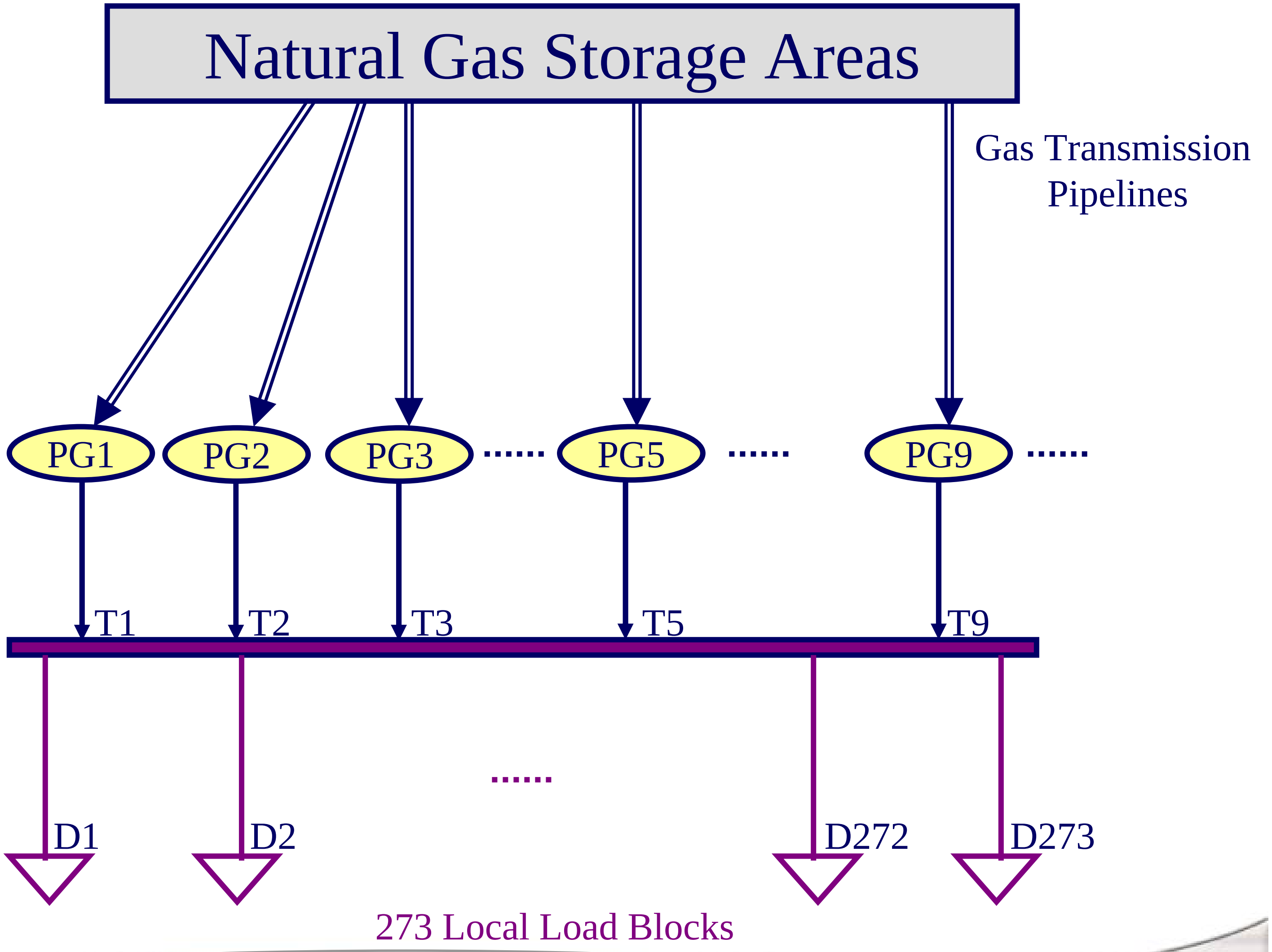
273 Local Load Blocks

Failures Consider in Centralized Test System

Generation units use natural gas cannot produce electricity

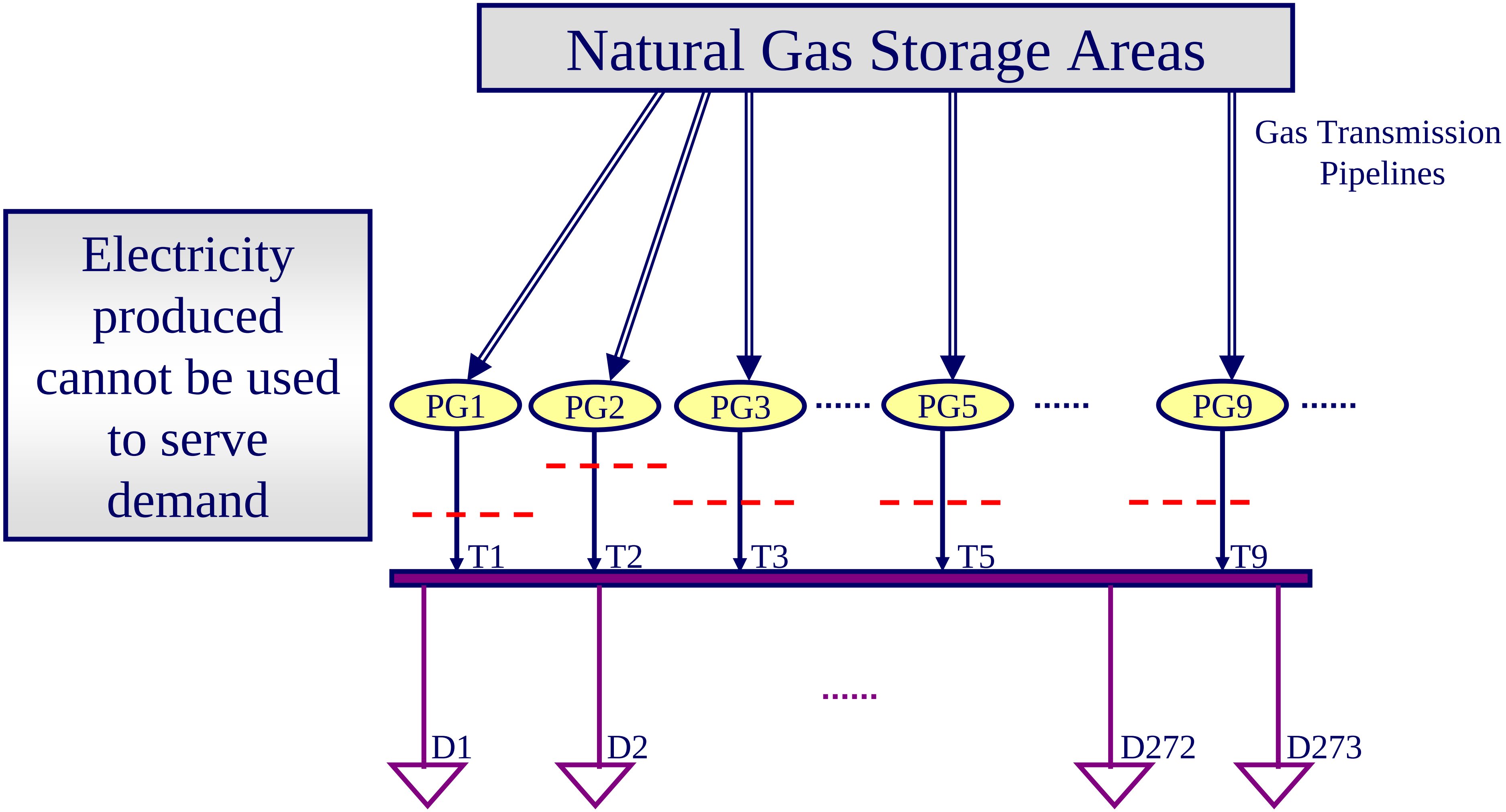


Failures Consider in Centralized Test System



Any generation can fail independent from each other

Failures Consider in Centralized Test System



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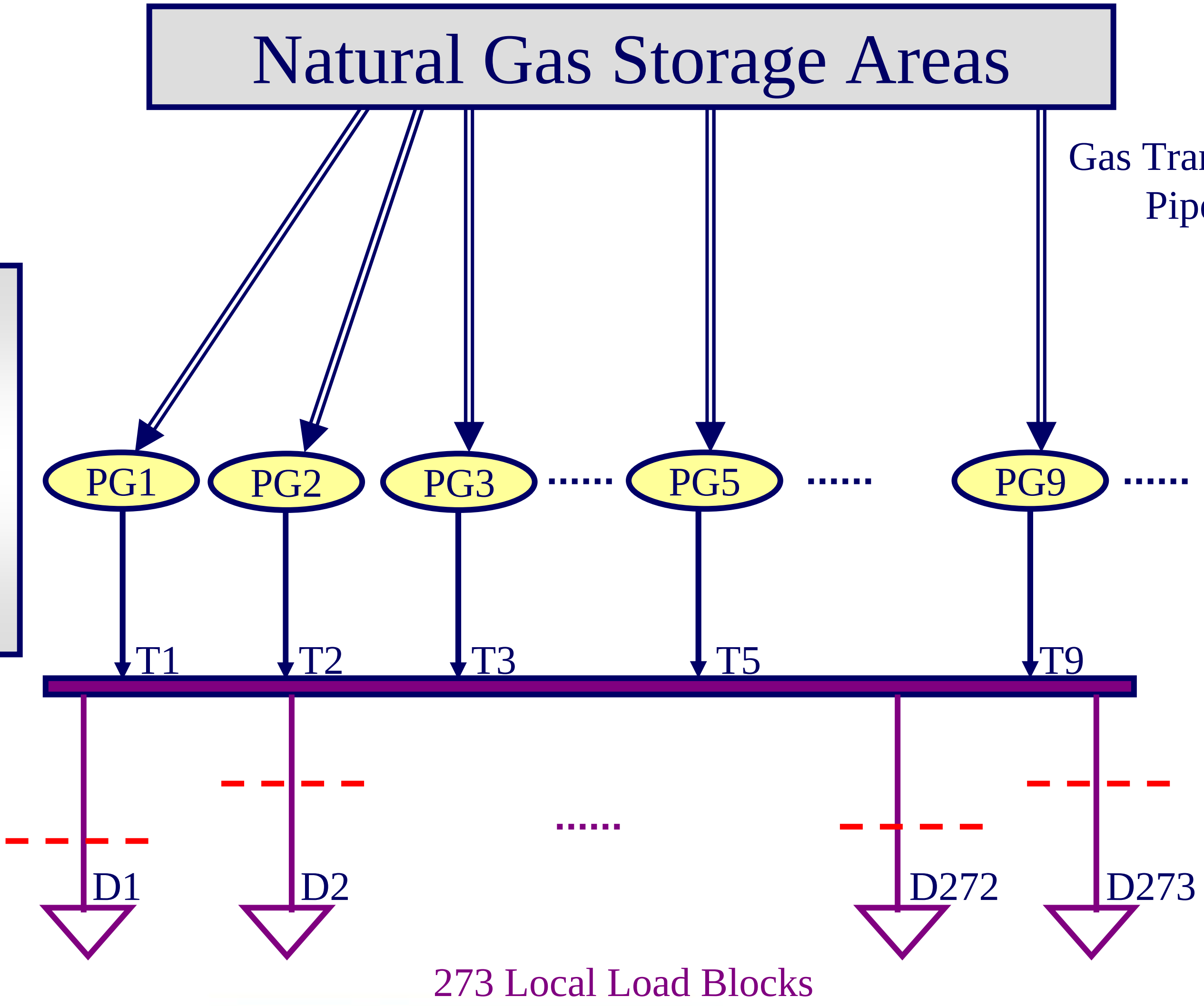
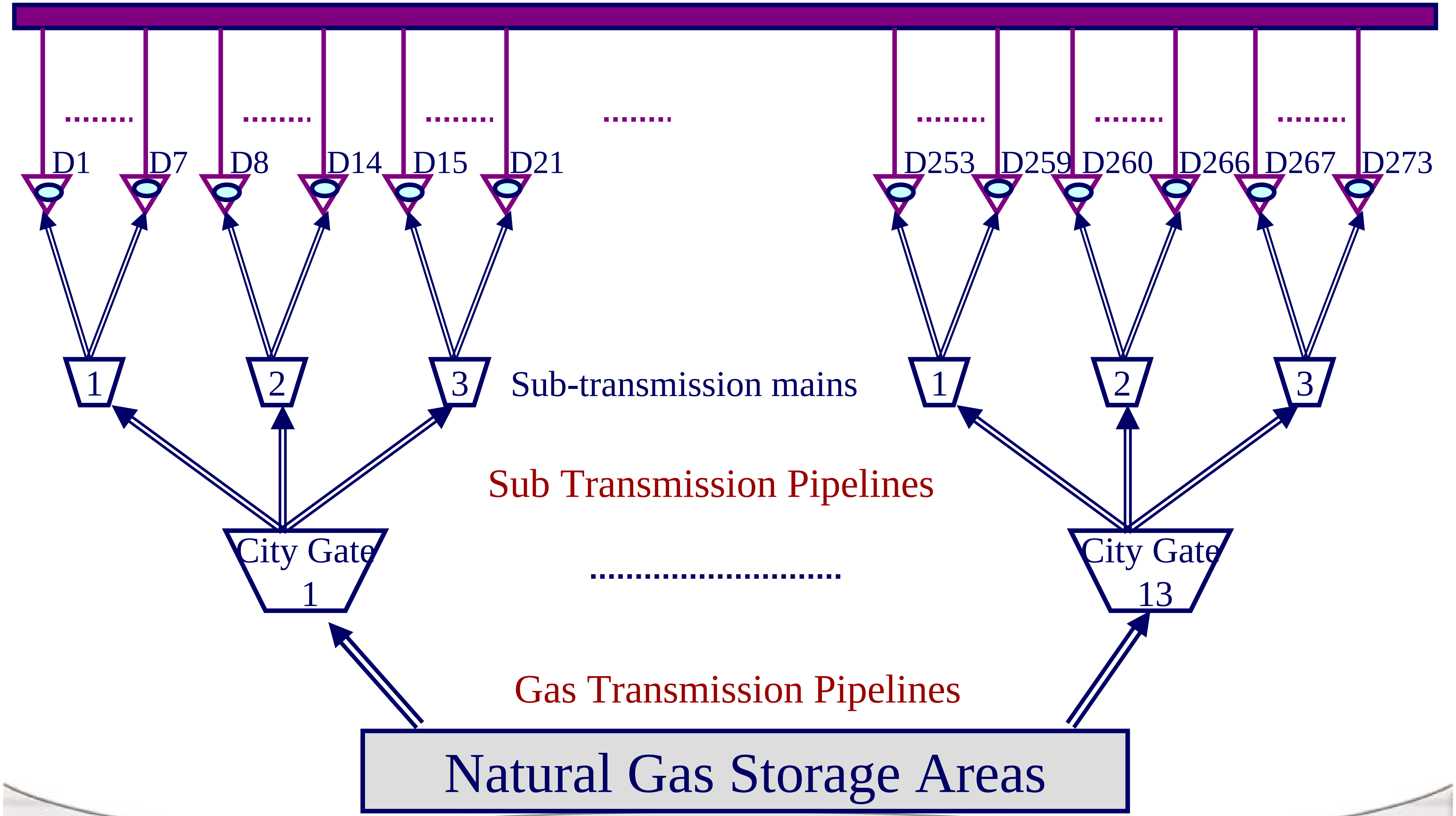


Demand in corresponding load block cannot be satisfied

Failures Considered in Distributed Test System



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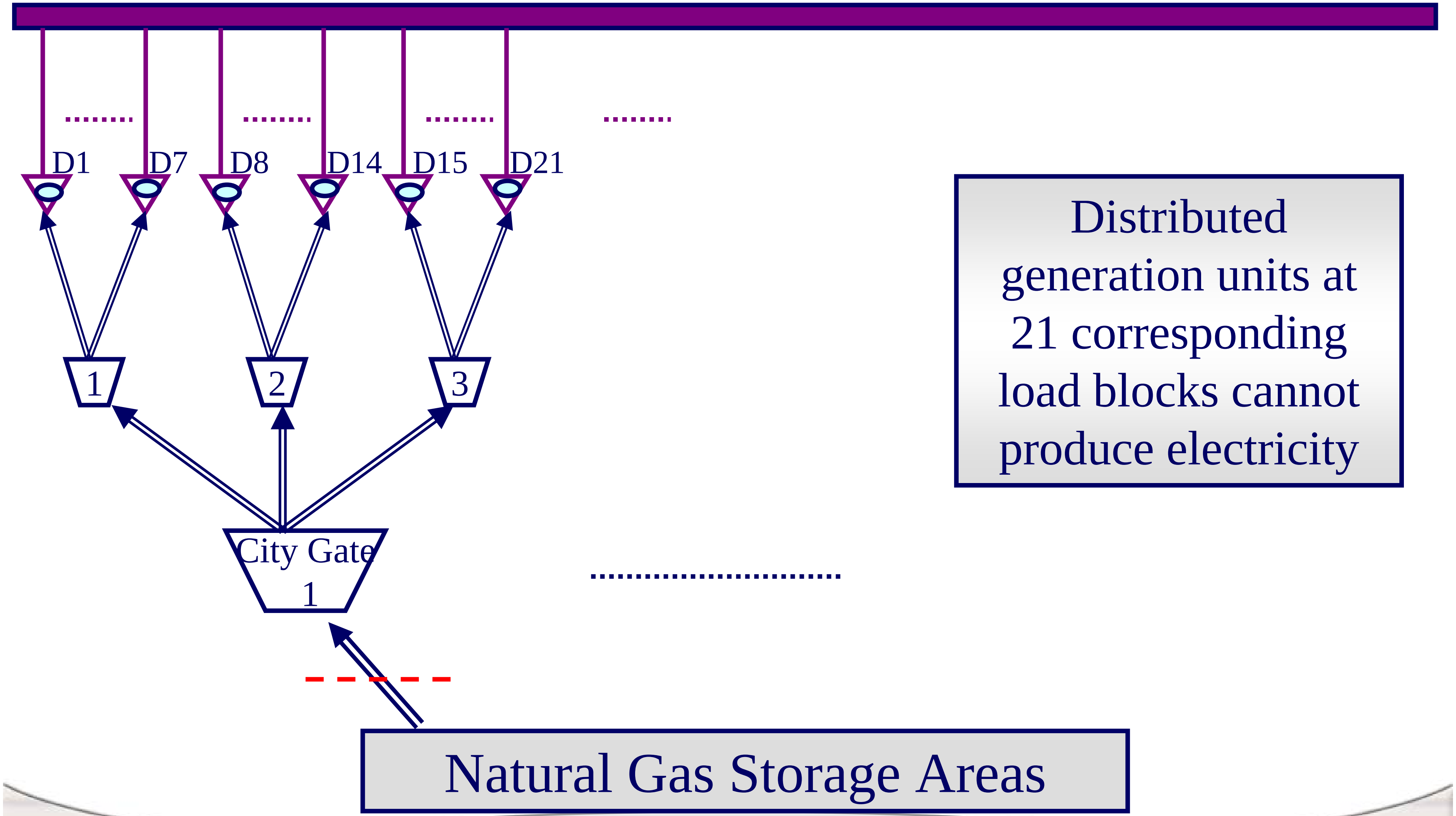
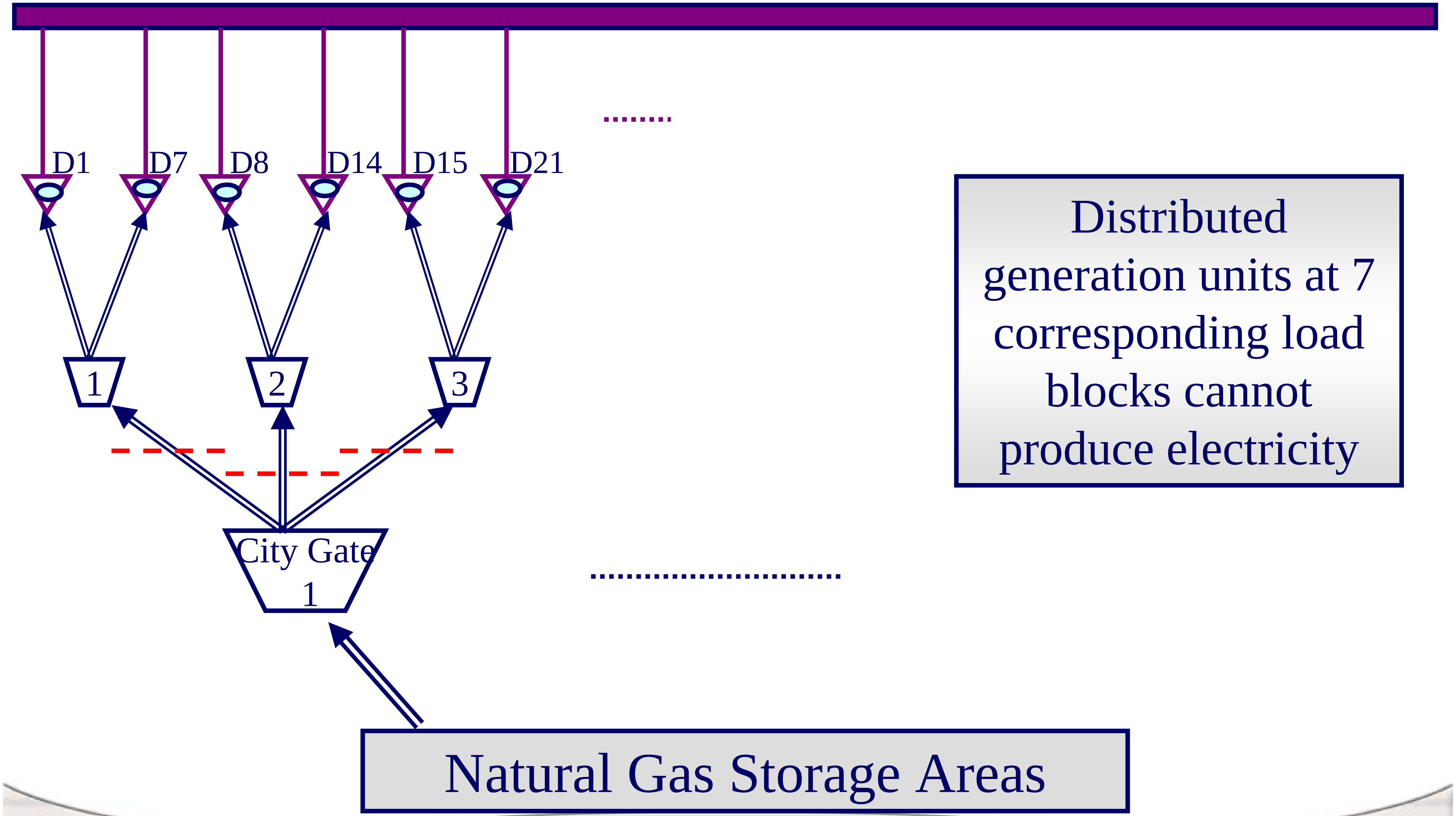


Distributed generation units at 21 corresponding load blocks cannot produce electricity

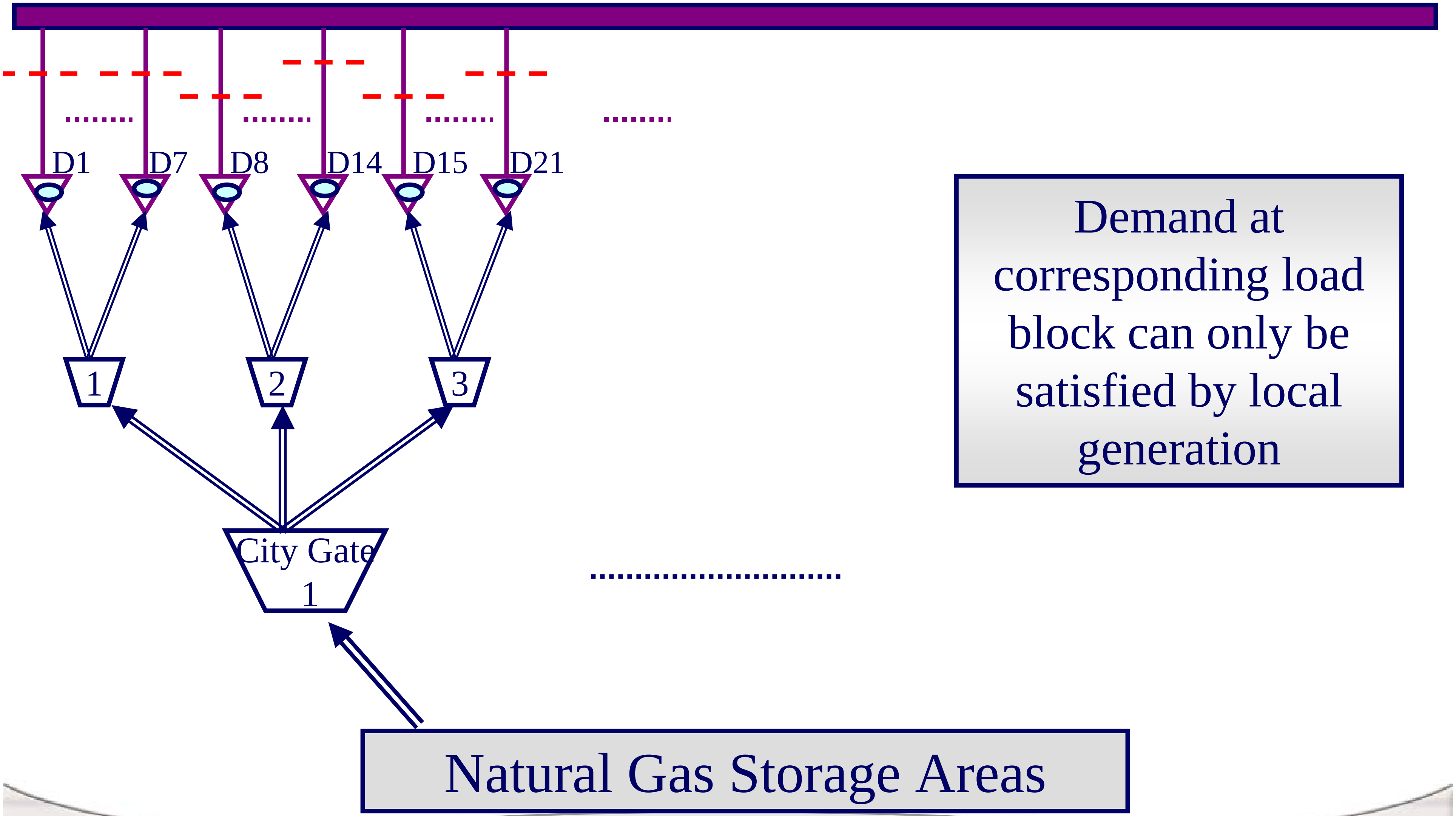
Natural Gas Storage Areas

Failures Considered in Distributed Test System



Distributed generation units at 7 corresponding load blocks cannot produce electricity

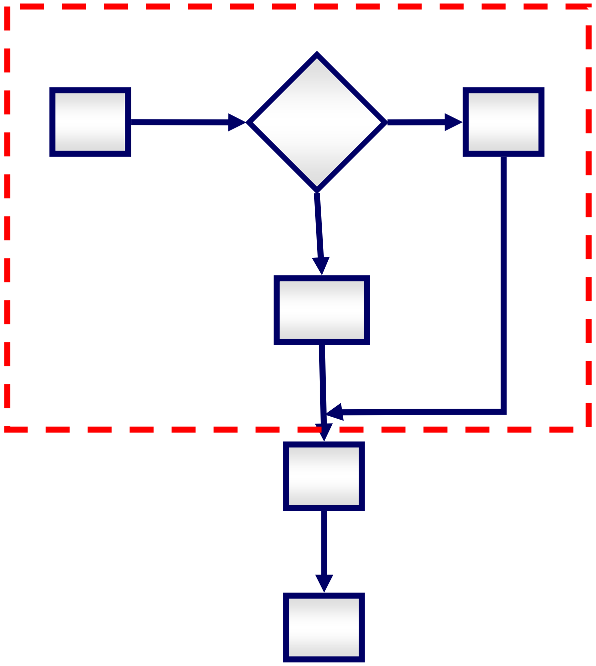
Failures Considered in Distributed Test System

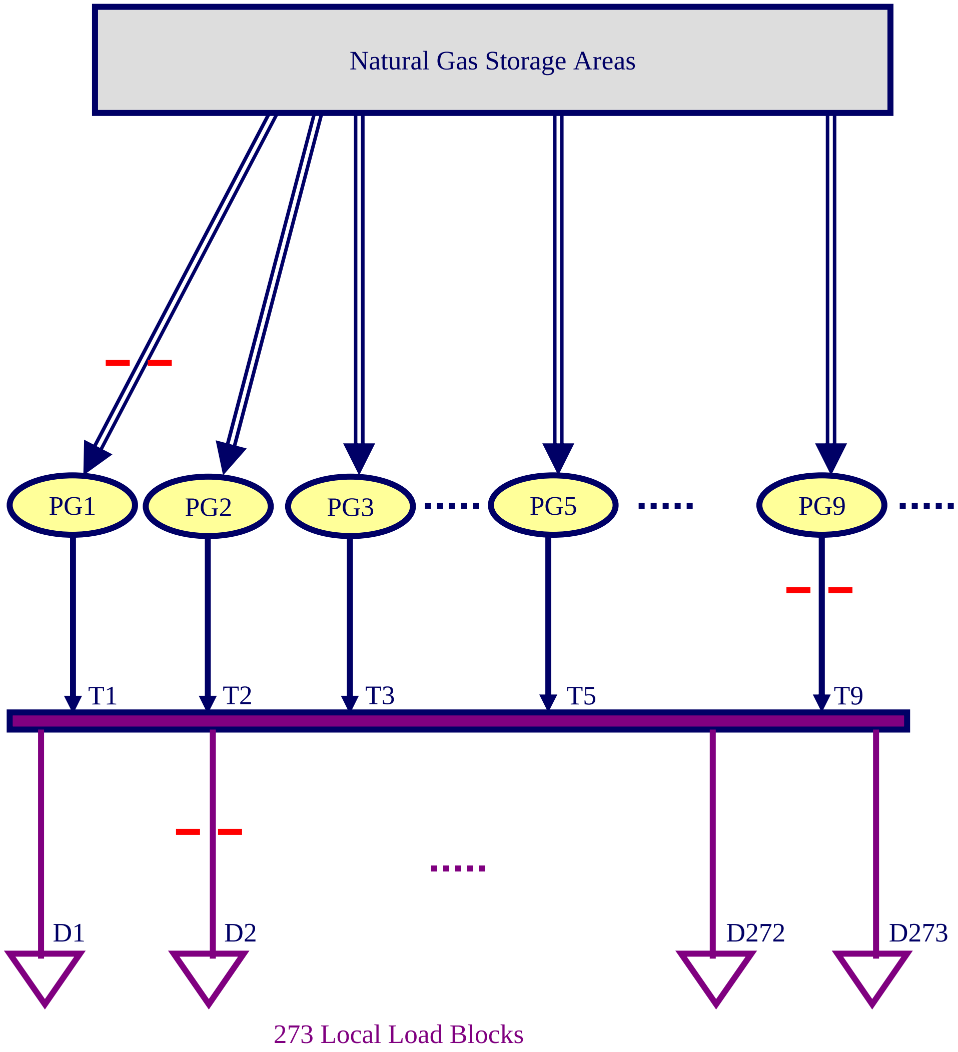


Demand at corresponding load block can only be satisfied by local generation

Natural Gas Storage Areas

Monte Carlo Simulation is used to generate *N* (10,000) Scenarios





Optimization Model

- Objective Function:
  - minimize cost (generation, expansion, unmet demand)
  - minimize NO_x
  - minimize CO_2 / SO_2
  - compromise or composite objective
- Problem constraints:
  - network topology
  - demand for power
  - power generation capacity
  - expansion locations
- Decision Variables
  - Generation: what units to use & when
  - Expansion: when & where to expand using what technology

$$\min \sum_{i=1}^{N} \sum_{k=1}^{K} x_{ik} c_k$$

$$s.t.$$

$$\sum_{i=1}^{K} x_{ik} \geq D_i \qquad \forall i$$

$$x_{ik} \geq 0 \qquad \forall i, k$$

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$$s.t.$$

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$$\boxed{x_{ik}} \geq 0 \qquad \forall i, k$$

We are Developing Multi-objective Stochastic Models

- Linear Programming
  - Single objective: minimize cost
  - Deterministic assumptions – generation units & transmission are always available or some multiple is available
- Stochastic programming
  - Uncertainty is explicitly considered
  - Two levels of decision variables:
    - variables in response to uncertainty (operations)
    - variables considering the distribution of uncertainty (expansion)
- Multiple-objective optimization
  - Simultaneously consider:
  - minimize cost (generation, expansion, unmet demand)
  - Minimize emissions (CO_2, SO_2, NO_x)
- A realistic & useful model must combine these approaches

Different Levels of Problem Complexity

- Model 0
  - Centralized system, no expansion considered, one period
- Model 1
  - Distributed system, no expansion considered, one period
- Model 2
  - Centralized system with distributed generation unit investment choices, one period
- Model 3
  - Centralized system with expansion decision over *n* period
- Complexity
  - Up to 1,000,000 decision variables and constraints
  - GAMS-CPLEX on Workstation

Model 0: Notation

x_{ik}: The amount (MW) of electricity produced by generation unit k to satisfy demand of i^{th} scenario

c_k: The cost of producing electricity (\$/MW) by generation unit k

IE_i: The amount (MW) of unmet electricity due to the insufficient electricity supply in scenario i

NSD_i: The amount (MW) of unmet electricity due to the failure of distribution lines in scenario i

f: The cost of unmet demand (\$/MW)

D_{i}: Servable demand (MW) in scenario I

Cap_{ik}: Available capacity of generation unit k in scenario i

Model 0
Centralized System, min Cost

$$\min \sum_{i=1}^{10,000} \sum_{k=1}^{K} x_{ik} c_k + \sum_{i=1}^{10,000} (IE_i + NSD_i) f$$

Model 0
Centralized System, min Cost

$$\min \sum_{i=1}^{10{,}000} \sum_{k=1}^{K} x_{ik} c_k + \sum_{i=1}^{10{,}000} (IE_i + NSD_i) f$$

Cost of unmet demand

Model 0

Centralized System, min Cost

$$\min \boxed{\sum_{i=1}^{10,000} \sum_{k=1}^{K} x_{ik} c_k} + \sum_{i=1}^{10,000} (IE_i + NSD_i) f$$

Cost of generation

Model 0
Centralized System, min Cost

$$\min \sum_{i=1}^{10,000} \sum_{k=1}^{K} x_{ik} c_k + \sum_{i=1}^{10,000} (IE_i + NSD_i) f$$

$$s.t.$$

$$IE_i + \sum_{i=1}^{K} x_{ik} \geq D_i \qquad \forall i$$

$$x_{ik} \leq Cap_{i,k} \qquad \forall i, k$$

$$x_{ik}, IE_i \geq 0 \qquad \forall i, k$$

Model 0
Centralized System, min Cost

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$$s.t.$$

$$IE_i + \sum_{i=1}^{K} x_{ik} \geq \boxed{D_i} \qquad \forall i$$

$$x_{ik} \leq \boxed{Cap_{i,k}} \qquad \forall i,k$$

$$x_{ik}, IE_i \geq 0 \qquad \forall i,k$$

Servable Demand

&

Capacity Available of generation unit k

For Scenario *i*

Model 1: Notation

G_{il}: The amount (MW) of electricity produced by distributed generation unit l to satisfy servable demand of i^{th} scenario

L_{il}: The amount (MW) of electricity produced by distributed generation unit l to satisfy local demand of i^{th} scenario

d_l : The cost of producing electricity (\$/MW) by distributed generation unit l

IE_i: The amount (MW) of unmet servable electricity due to the insufficient electricity supply in scenario i

LIE_{il}: The amount (MW) of unmet local electricity due to the insufficient electricity supply in scenario i

f: The cost of unmet demand (\$/MW)

D_i: Servable demand (MW) in scenario i

LD_{il}: Local demand (MW) in scenario i at load block l

$CapDW_{il}$: Available capacity of generation unit l in scenario i where distribution line is working

$CapDF_{il}$: Available capacity of generation unit l in scenario i where distribution line is failed

p: Percentage of steam used; r: revenue obtained from steam (\$/MW)

Model 1
Distributed System: min Cost

$$\min \sum_{i=1}^{10,000} \sum_{l=1}^{273} (G_{il} + L_{il}) d_l + \sum_{i=1}^{10,000} (IE_i + LIE_{il}) f - \sum_{i=1}^{10,000} \sum_{l=1}^{273} \boxed{(G_{il} + L_{il}) pr}$$

$st.$

$$IE_i + \sum_{l=1}^{273} G_{il} \geq D_i \qquad \forall i$$

$$LIE_{il} + L_{il} \geq LD_{il} \qquad \forall i, l$$

$$G_{il} \leq CapDW_{il} \qquad \forall i, l$$

$$L_{il} \leq CapDF_{il} \qquad \forall i, l$$

$$G_{il}, L_{il} \geq 0 \qquad \forall i, l$$

Profit due to cogeneration capability

Model 1
Distributed System: min Cost

$$\min \sum_{i=1}^{10,000}\sum_{l=1}^{273}(G_{il}+L_{il})d_l + \sum_{i=1}^{10,000}(IE_i + LIE_{il})f - \sum_{i=1}^{10,000}\sum_{l=1}^{273}(G_{il}+L_{il})pr$$

$st.$

$$IE_i + \sum_{l=1}^{273} G_{il} \ge D_i \qquad \forall i$$

$$LIE_{il} + L_{il} \ge LD_{il} \qquad \forall i,l$$

$$G_{il} \le CapDW_{il} \qquad \forall i,l$$

$$L_{il} \le CapDF_{il} \qquad \forall i,l$$

$$G_{il}, L_{il} \ge 0 \qquad \forall i,l$$

Cost Trade-off between Centralized vs. Distributed System

	Centralized Test System	Distributed Test System
Electricity Generation Cost	$ 67,000,000	$ 438,000,000
Demand Not Satisfied	141,00 MW	23MW
Cost of Unsatisfied Demand	$ 1,416,000,000	$230,000
Steam Revenue	0	130,000,000
Operation Cost	$ 1,483,000,000	$ 308,000,000

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Only 100 MW due to insufficient energy supply

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Moving from centralized to distributed can provide cost benefit

Optimization Considers Different Objectives

CO$_2$

$$\min \sum_{i=1}^{10,000} \sum_{k=1}^{K} x_{ik} CO2_k + \sum_{i=1}^{10,000} (IE_i + NSD_i) w$$

SO$_2$

$$\min \sum_{i=1}^{10,000} \sum_{k=1}^{K} x_{ik} SO2_k + \sum_{i=1}^{10,000} (IE_i + NSD_i) w$$

NO$_X$

$$\min \sum_{i=1}^{10,000} \sum_{k=1}^{K} x_{ik} NOX_k + \sum_{i=1}^{10,000} (IE_i + NSD_i) w$$

Multi-Objective

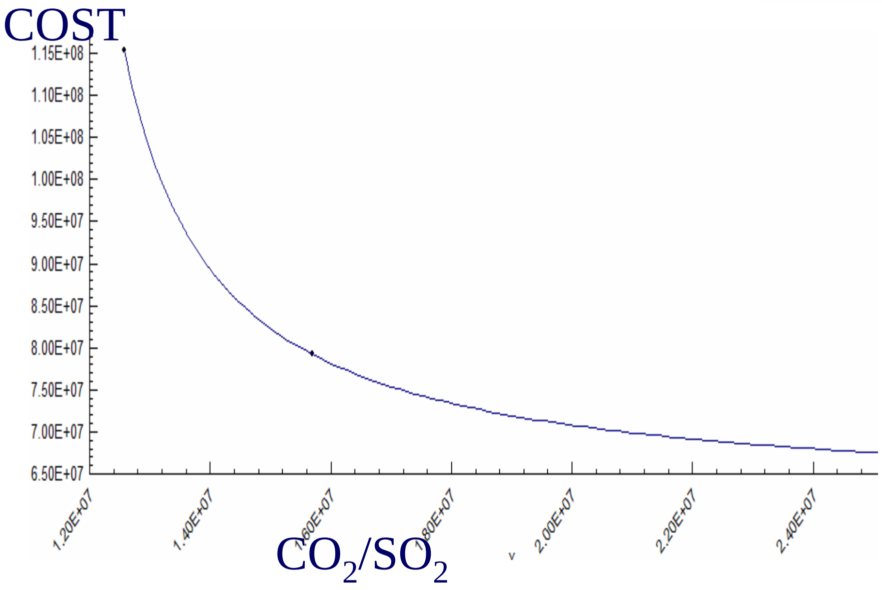
$$\min \sum_{i=1}^{10,000} \sum_{k=1}^{K} w_1 x_{ik} \alpha_1 c_k + \sum_{i=1}^{10,000} (IE_i + NSD_i) \alpha f$$

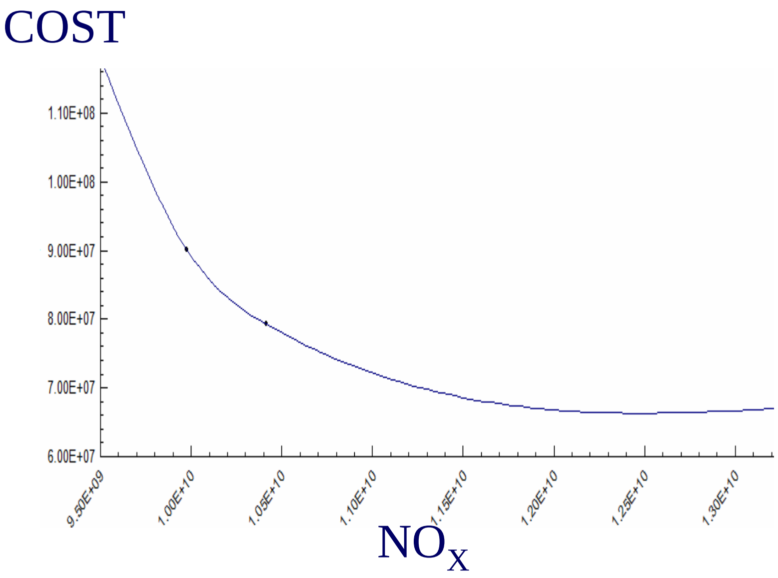
$$\sum_{i=1}^{10,000} \sum_{k=1}^{K} w_2 x_{ik} \alpha_2 CO2_k + \sum_{i=1}^{10,000} \sum_{k=1}^{K} w_3 x_{ik} \alpha_3 SO2_k + \sum_{i=1}^{10,000} \sum_{k=1}^{K} w_4 x_{ik} \alpha_4 NOX_k$$

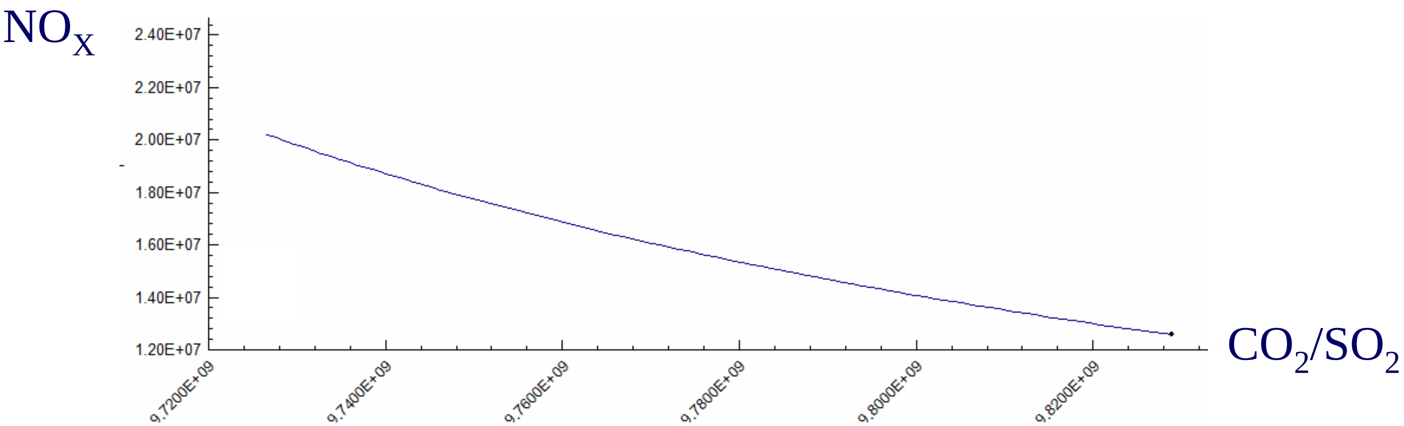
Optimum Solution with each objective function in centralized system

Objective Function	min cost	min CO_2	min SO_2	min NO_X
Generation Cost ($)	67,000,000	117,000,000	117,000,000	115,000,000
CO_2 (Lbs)	13,373,000,000	9,501,000,000	9,501,000,000	9,828,000,000
SO_2 (Lbs)	92,000,000	27,000,000	27,000,000	33,000,000
NO_X (Lbs)	25,000,000	23,000,000	23,000,000	12,000,000

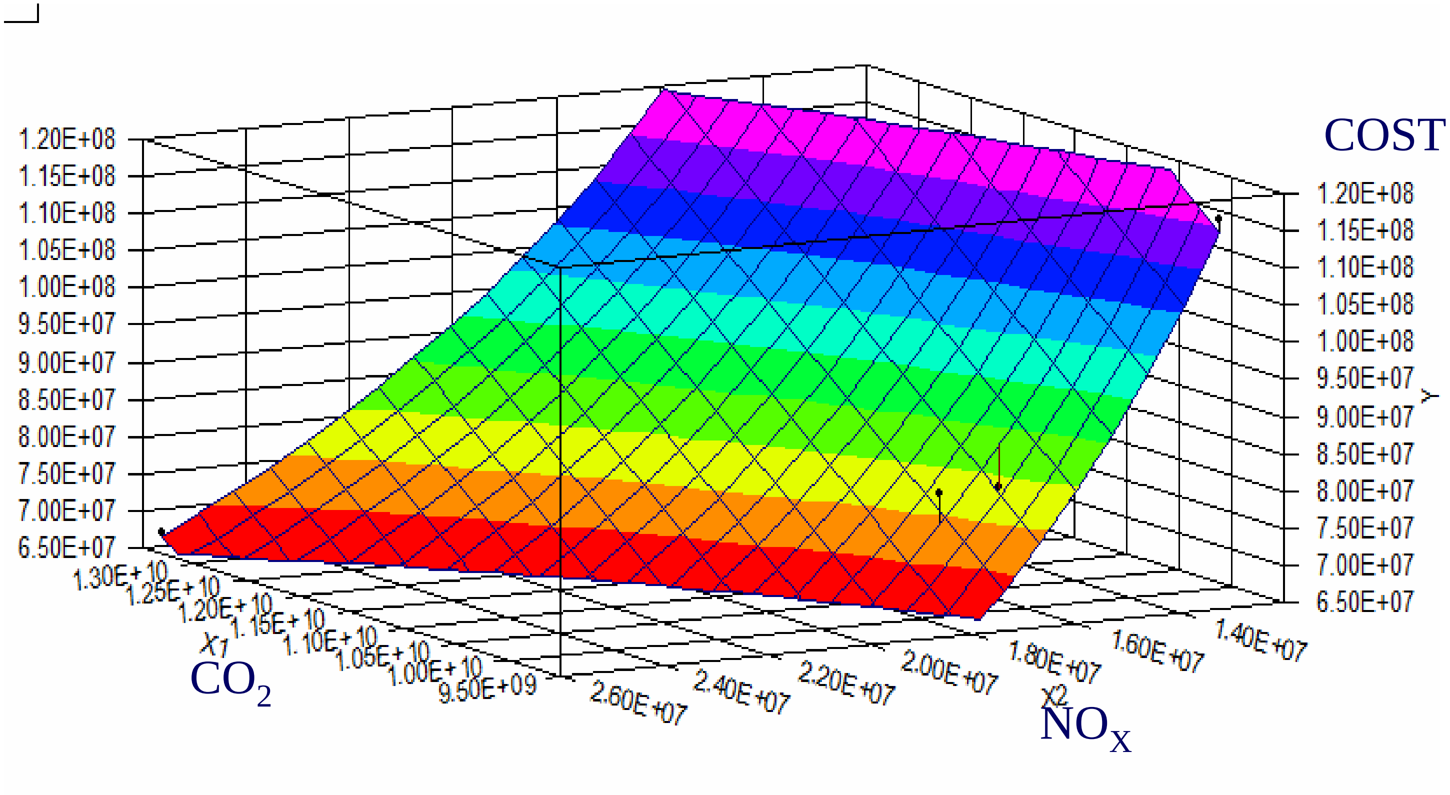
Trade-off Analyses for Centralized System







Trade-off Analysis with Three Objectives



Model 3: Notation

x_{tik}: The amount (MW) of electricity produced by generation unit l to satisfy servable demand of i^{th} scenario in period t

y_{tiq}: The amount (MW) of electricity produced by new generation unit q to satisfy servable demand of i^{th} scenario in period t

G_{til}: The amount (MW) of electricity produced by distributed generation unit l to satisfy servable demand of i^{th} scenario in period t

L_{til}: The amount (MW) of electricity produced by distributed generation unit l to satisfy local demand of i^{th} scenario in period t

IE_{ti}: The amount (MW) of unmet servable electricity due to the insufficient electricity supply in scenario i in period t

LIE_{til}: The amount (MW) of unmet local electricity due to the insufficient electricity supply in scenario i in period t

u_{tq}: 1 if central generation unit q is built in period t, 0 otherwise

w_{tl}: 1 if distributed generation unit l is built in period t, 0 otherwise

D_{ti}: Servable demand for scenario i in period t

LD_{til}: Servable demand for scenario i at load block l in period t

Model 3: Notation

Cap_{tik}: Available capacity of generation unit k in scenario i in period t

$CapN_{tiq}$: Available capacity of new central unit q in scenario i in period t

$CapDW_{til}$: Available capacity of generation unit l in scenario i in period t where distribution line is working

$CapDF_{til}$: Available capacity of generation unit l in scenario i in period t where distribution line is failed

c_{tk}: The cost of producing electricity by central unit k in period t

e_{tq}: The cost of producing electricity by new central unit q in period t

d_{tl}: The cost of producing electricity by distributed unit l in period t

a_{tq}: The cost of building central unit q in period t

b_{tl}: The cost of building distributed unit l in period t

f_t: The cost of unmet demand in period t

p_t: The percentage of steam used in period t

r_t: The revenue obtained from steam in period t

Model 3
Centralized System with Expansion Possibilities over *n* Time Periods

$$\min \sum_{t=1}^{n}\sum_{i=1}^{10{,}000}\sum_{k=1}^{K} x_{tik}c_{tk} + \sum_{t=1}^{n}\sum_{i=1}^{10{,}000}\sum_{q=1}^{Q} y_{tiq}e_{tq} + \sum_{t=1}^{n}\sum_{q=1}^{Q} u_{tq}a_{tq} + \sum_{t=1}^{n}\sum_{i=1}^{10{,}000}\sum_{l=1}^{\max,l}(G_{til}+L_{til})d_{tl} + \sum_{t=1}^{n}\sum_{l=1}^{\max,l} w_{tl}b_{tl} +$$

$$\sum_{t=1}^{n}\sum_{i=1}^{10{,}000} IE_{ti}\,f_t + \sum_{t=1}^{n}\sum_{i=1}^{10{,}000}\sum_{l=1}^{\max,l} LIE_{til}\,f_t - \sum_{t=1}^{n}\sum_{i=1}^{10{,}000}\sum_{l=1}^{\max,l}(G_{til}+L_{til})\,p_t r_t$$

$st.$

$$IE_{ti} + \sum_{k=1}^{K} x_{tik} + \sum_{q=1}^{Q} y_{tiq} + \sum_{l=1}^{\max,l} G_{til} \ge D_{ti} \qquad \forall t,i$$

$$LIE_{til} + L_{til} \ge LD_{til} \qquad \forall t,i,l$$

$$x_{tik} \le Cap_{tik} \qquad \forall t,i,k$$

$$y_{tiq} \le CapN_{tiq}\sum_{\tau=1}^{t} u_{\tau q} \qquad \forall t,i,q$$

Model 3

Centralized System with Expansion Possibilities over *n* Time Periods

$$G_{til} \leq CapDW_{til} \sum_{\tau=1}^{t} w_{\tau l} \qquad \forall t,i,l$$

$$L_{til} \leq CapDF_{til} \sum_{\tau=1}^{t} w_{\tau l} \qquad \forall t,i,l$$

$$\sum_{t=1}^{n} w_{tl} = 1 \qquad \forall l$$

$$\sum_{t=1}^{n} u_{tq} = 1 \qquad \forall l$$

$$w_{tl} \in \{0,1) \qquad \forall t,l \qquad u_{tq} \in \{0,1) \qquad \forall t,q$$

$$x_{tik} \geq 0 \qquad \forall t,i,k \qquad G_{til}, L_{til}, LIE_{til} \geq 0 \qquad \forall t,i,l \qquad IE_{ti} \geq 0 \qquad \forall t,i$$

THANK YOU

Model 2: Notation

x_{ik}: The amount (MW) of electricity produced by generation unit l to satisfy servable demand of i^{th} scenario

G_{il}: The amount (MW) of electricity produced by distributed generation unit l to satisfy servable demand of i^{th} scenario

L_{il}: The amount (MW) of electricity produced by distributed generation unit l to satisfy local demand of i^{th} scenario

IE_i: The amount (MW) of unmet servable electricity due to the insufficient electricity supply in scenario i

LIE_{il}: The amount (MW) of unmet local electricity due to the insufficient electricity supply in scenario i

w_l: 1 if distributed generation unit l is built, 0 otherwise

D_i: Servable demand for scenario i

LD_{il}: Servable demand for scenario i at load block l

Model 2: Notation

Cap_{ik}: Available capacity of generation unit k in scenario i

$CapDW_{il}$: Available capacity of generation unit l in scenario i where distribution line is working

$CapDF_{il}$: Available capacity of generation unit l in scenario i where distribution line is failed

c_k: The cost of producing electricity by central unit k

d_l: The cost of producing electricity by distributed unit l

b_l: The cost of building distributed unit l

f: The cost of unmet demand

p: The percentage of steam used

r: The revenue obtained from steam

Model 2

Centralized with distributed generation expansion

$$\min \sum_{i=1}^{10,000} \sum_{k=1}^{K} x_{ik} c_k + \sum_{i=1}^{10,000} \sum_{l=1}^{\max,l} (G_{il} + L_{il}) d_l + \sum_{l=1}^{\max,l} w_l b_l +$$

$$\sum_{i=1}^{10,000} IE_i f + \sum_{i=1}^{10,000} \sum_{l=1}^{\max,l} LIE_{i,l} f - \sum_{i=1}^{10,000} \sum_{l=1}^{\max,l} (G_{il} + L_{il}) pr$$

$st.$

$$IE_i + \sum_{k=1}^{K} x_{ik} + \sum_{l=1}^{\max,l} G_{il} \geq D_i \qquad \forall i$$

$$LIE_{il} + L_{il} \geq LD_{il} \qquad \forall i, l$$

$$x_{ik} \leq Cap_{ik} \qquad \forall i, k$$

$$G_{il} \leq CapDW_{il} w_l \qquad \forall i, l$$

$$L_{i,l} \leq CapDF_{il} w_l \qquad \forall i, l$$

$$w_l \in \{0,1) \qquad \forall l$$

$$x_{ik} \geq 0 \qquad \forall i, k \qquad G_{il}, L_{il}, LIE_{il} \geq 0 \qquad \forall i, l \qquad IE_i \geq 0 \qquad \forall i$$

Model 2

Centralized with distributed generation expansion

$$\min \sum_{i=1}^{10,000}\sum_{k=1}^{K} x_{ik}c_k + \sum_{i=1}^{10,000}\sum_{l=1}^{\max,l}(G_{i,l}+L_{il})d_l + \sum_{l=1}^{\max,l} w_l b_l +$$

$$\sum_{i=1}^{10,000} IE_i f + \sum_{i=1}^{10,000}\sum_{l=1}^{\max,l} LIE_{i,l} f - \sum_{i=1}^{10,000}\sum_{l=1}^{\max,l}(G_{i,l}+L_{i,l})pr$$

min CO2,SO2, NOX or combination of them can easily be modeled. Renewable energy sources can benefit for this objectives.

$$LIE_{il} + L_{il} \geq LD_{i,l} \qquad \forall i,l$$

$$x_{ik} \leq Cap_{ik} \qquad \forall i,k$$

$$G_{il} \leq CapDW_{il} w_l \qquad \forall i,l$$

$$L_{i,l} \leq CapDF_{il} w_l \qquad \forall i,l$$

$$w_l \in \{0,1) \qquad \forall l$$

$$x_{ik} \geq 0 \qquad \forall i,k \qquad G_{il}, L_{il}, LIE_{il} \geq 0 \qquad \forall i,l \qquad IE_i \geq 0 \qquad \forall i$$

Centralized with Distributed Generation vs. Distributed System Total Cost

	Centralized Test System	Centralized with Expansion by Distributed Units
Electricity Generation Cost	$ 67,000,000	$69,000,000
Demand Not Satisfied	141,000 MW	115,000 MW
Cost of Unsatisfied Demand	$ 1,416,000,000	$1,159,000,000
Steam Revenue		$1,000,000
Building Cost		$75,000,000
Building cost	$ 1,483,000,000	$ 1,302,000,000

Centralized with Distributed Generation vs. Distributed System Total Cost

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Due to distribution line failure

Centralized with Distributed Generation vs. Distributed System Total Cost

	Centralized Test System	Centralized with Expansion by Distributed Units
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Expansion with distributed generation units benefits